A brief visit to Madrid for supplies, and to see the Astronomical Observatory, followed, and finally I returned to Daroca August 13. Observations will be continued regularly till the day following the eclipse, August 31, when the various pieces of apparatus will be gradually collected from the several places and put aboard the *Cæsar*, which will sail from Nice on September 13 for the United States.

The climate in Spain has proved to be unexpectedly agreeable and favorable for such an eclipse expedition as this. The air is generally dry and for the most part cloudless; there have been no thunderstorms and no rains for weeks; the temperature at Daroca, 2200 feet above the sea, reaches about 90° F. at midday, but falls to 50° or 60° every night. The extreme heat was 95° on two days, and the lowest night temperature 42°. It should be noted that in the daytime the relative humidity falls to about 40 per cent on the average, but on the hottest days to 25 or 30 per cent, while at night it rises to above 90 per cent, often to 96 per cent. This wide range of temperature and relative humidity is accompanied by a remarkably steady barometer, ranging about two-tenths of an inch per day, and only four-tenths of an inch during several weeks. The cyclonic system that prevails in the United States does not seem to exist in Spain in the summer. On the sea coast the sea breeze is especially vigorous, so that the stations Porta Coeli, Castellon, and Tortosa will record that feature fully in connection with the eclipse, while the inland stations Daroca, Zaragoza, and Guadalajara, will be free from it. This will enable us to study the so-called "eclipse cyclone" with data bearing directly upon the subject. The circulars containing instructions regarding the shadow bands have been distributed very widely and there is an apparent interest in this subject.

The Spanish people have been most hospitable toward the American eclipse parties, and indeed to all the visiting scientists, of whom there are now many in Spain, and they have always most cordially assisted in carrying out the plans proposed for the benefit of the expeditions. We shall always recall their hospitality with feelings of gratitude and obligation.

DAROCA, SPAIN, *August 30, 1905.*

The total eclipse was an entire success at this station so far as the weather was concerned. The 29th had been a very anxious day, because it was heavily clouded, with occasional showers, for the first time in four weeks, but the wind shifted from west to north toward evening, the temperature dropped to 40° during the night and the morning of the 30th was fair. There were numerous cumulus and alto-cumulus clouds, but they gradually dissipated by noon, and at 1 p. m. the region around the sun was exceptionally clear. The sky polarization was 58 per cent during the eclipse, and while we have had occasional readings of 72 per cent, this means an atmosphere quite clear of uncondensed vapor. The barometer remained unaffected, the temperature dropped 8° F. in the shade and 18° in the sunshine. The wind was very light and there seemed to be no special change during the middle hour of the eclipse. The shadow bands were very feeble and disappointing, but they were seen for 40 seconds ending within 15 seconds of the second contact, and again less distinctly for a half a minute from about 30 seconds after the third contact. Their direction of motion was along the central line and they lay exactly perpendicular to it; their width was such as to give about two bands and three spaces to the foot; the velocity was about three feet per second. The electrical observations were carried on without interruption from 6 a. m. till 8 p. m., but the results can not be stated without computation. The corona as seen through the opera glass and in my 3.5-inch telescope was a beautiful sight, as usual, typical of the corona at the maximum of the sun-spot period. Two common sized spots and

three smaller ones were present, and the times of contacts I and IV and the passage of the limb of the moon over these spots were noted at least approximately. The rays of the corona were generally radial, the polar curved rays being obscure and irregular. There were several stellar points on the streamers, which extended generally two diameters from the sun. There were no very extensive streamers, but the corona was bright and rather condensed, of a steel gray, pearly color, as seen in some electrical phenomena; the inner corona was brilliant and there were two long groups of superb rosy-tinted prominences which will become famous in the history of solar eclipses. The work with the spectroscopes and cameras at Daroca is believed to have been of the best quality, but it is not known at this time what special information was secured. The health of the entire party of twenty persons has been excellent.

OBSERVATIONS OF EARTH TEMPERATURE IN JAPAN.[1]

By Dr. S. TETSU TAMURA. Dated Washington, D. C., June 10, 1905.

(1) INTRODUCTION.

The earth's crust and the outer soil furnish examples of the periodic flow of heat that illustrate Fourier's beautiful theorem. If the earth's surface be heated and cooled periodically, a thermometer sunk in the ground will exhibit corresponding variations of temperature. By day the surface of the earth is heated, a diurnal temperature wave is propagated into the interior and the indication of the thermometer gradually rises. As the earth's surface is cooled at night, the thermometer will exhibit a fall of temperature. If, therefore, the surface temperature is a periodic function of the time, then the temperature at any depth will vary in a corresponding periodic manner. When the periodic variation has been maintained for a sufficient time, the oscillations of temperature, at any depth, will attain a fixed character so that the mean temperature at each point remains steady. There are also annual temperature waves due to heating during summer and cooling during winter, and irregular oscillations, due to cloudiness, rainfall, snow on ground, etc.

If θ is the temperature of the soil at any depth x, t the time, and a^2 the diffusivity[2] of the soil, we have the equation:

$$\frac{\partial \theta}{\partial t} = a^2 \frac{\partial^2 \theta}{\partial x^2}. \tag{1}$$

If θ is a periodic function of the time t, or $\theta = F(t)$ for the earth's surface $x = 0$, we can integrate the above equation readily. As the equation (1) is linear with a constant coefficient we get a particular solution by the following device used in articles 7 and 8 of Byerly's, Fourier's Series and Spherical Harmonics. Let $\theta = e^{\beta t + ax}$

and substitute this in (1): we obtain $\beta = a^2 a^2$

whence $\theta = e^{\beta t \pm \frac{x}{a} \sqrt{\beta}} \tag{2}$

which is a solution of equation (1) no matter what value is given to β.

In order to put this into a more convenient trigonometric form, replace β by $\pm \beta i$ and equation (2) becomes

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[1] More than a year ago Doctor Tamura consented to give us some account of work done by Japanese meteorologists, which promise, however, could not be fulfilled until to-day, on account of the pressure of his other work. The paper here given is a review of memoirs on the earth-temperature observations by his compatriots, Doctors Nakamura, Oishi, and Okada. Doctor Tamura has added, as an explanation, a note on the mathematical principle of the problems of earth temperature that will facilitate the reading by those who are not familiar with the subject.—[ED.]

[2] The diffusivity of the soil $a^2 = \dfrac{k}{\rho c}$ where $k =$ the conductivity of the soil, ρ its density, and c its specific heat. These four quantities are assumed constant in the mathematical analysis, but actually they vary with the water in the soil and also with the distance below the surface.

$$\theta = e^{\pm\,\beta t i \,\pm\, \frac{x}{a}\sqrt{\beta}\,\sqrt{\pm i}}$$

whence, by the relations

$$\sqrt{i} = \pm \frac{1}{2}\sqrt{2}\,(1+i)$$

$$\sqrt{-i} = \pm \frac{1}{2}\sqrt{2}\,(1-i)$$

we get

$$\theta = e^{\pm\,\beta t i \,\pm\, \frac{x}{a}(1\pm i)\sqrt{\beta/2}}$$

or

$$\theta = e^{\left(\beta t - \frac{x}{a}\right)i}\, e^{-\frac{x}{a}\sqrt{\beta/2}} \tag{3}$$

and

$$\theta = e^{-\left(\beta t - \frac{x}{a}\right)i}\, e^{-\frac{x}{a}\sqrt{\beta/2}}. \tag{4}$$

By adding these values of θ and dividing by 2, we have [3]

$$\theta = e^{-\frac{x}{a}\sqrt{\beta/2}} \cos\left(\beta t - \frac{x}{a}\sqrt{\beta/2}\right) \tag{5}$$

By subtracting (3) from (4) and dividing by $2i$, we obtain

$$\theta = e^{-\frac{x}{a}\sqrt{\beta/2}} \sin\left(\beta t - \frac{x}{a}\sqrt{\beta/2}\right) \tag{6}$$

In similar manner we can build up the expressions

$$\theta = e^{+\frac{x}{a}\sqrt{\beta/2}} \cos\left(\beta t + \frac{x}{a}\sqrt{\beta/2}\right) \tag{7}$$

$$\theta = e^{+\frac{x}{a}\sqrt{\beta/2}} \sin\left(\beta t + \frac{x}{a}\sqrt{\beta/2}\right) \tag{8}$$

All these expressions (3)–(8) are particular solutions of equation (1); but (7) and (8) do not satisfy our fundamental condition, for according to them θ becomes greater as x is greater. Equations (5) and (6) become more general by introducing certain constants, A, q, and C; these do not vary with time, but only with depth, and we can write the equations as follows:

$$\theta = A\, e^{-\frac{x}{a}\sqrt{\beta/2}} \sin\left(\beta t - \frac{x}{a}\sqrt{\frac{\beta}{2}} + q\right) + C \tag{9}$$

$$\theta = A\, e^{-\frac{x}{a}\sqrt{\beta/2}} \cos\left(\beta t - \frac{x}{a}\sqrt{\frac{\beta}{2}} + q\right) + C. \tag{10}$$

We can build up from equation (9) a general solution,

$$\theta = \sum_{n=1}^{n=\infty} A_n\, e^{-\frac{x}{a}\sqrt{n\omega/2}} \sin\left(n\omega t - \frac{x}{a}\sqrt{\frac{n\omega}{2}} + q_n\right). \tag{11}$$

and a similar cosine series from (10), where n is any positive whole number from one to infinity.

It is clear that if we confine our attention to a definite point of the soil, so that x remains constant while t varies, then θ will vary periodically, returning to the same value when t is increased by any multiple of $\frac{2\pi}{\beta}$.

Hence, if it be assumed that the earth's surface is subjected to a simple harmonic change of temperature, and that the complete period of a heating and cooling is T, we have

$$\frac{2\pi}{\beta} = T$$

or

$$\beta = \frac{2\pi}{T} \tag{12}$$

and consequently β measures the rapidity of the alternation of temperature. Then (9) becomes

$$\theta = A\, e^{-\frac{x}{a}\sqrt{\pi/T}} \sin\left(\frac{2\pi}{T} t - \frac{x}{a}\sqrt{\frac{\pi}{T}} + q\right) + C. \tag{13}$$

[3] Byerly's Integral Calculus, article 35. Fourier's Series and Spherical Harmonics, articles 49 and 52.

Let us examine the time at which a thermometer placed in the soil at a depth, x, below the earth's surface will reach its highest or lowest temperature. This will happen when

$$\sin\left(\frac{2\pi}{T} t - \frac{x}{a}\sqrt{\frac{\pi}{T}} + q\right)$$

attains its greatest or least value, that is, when the angle is $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$, etc., or when the sine is ± 1; or, in other words, the temperature will be maximum at the depth x_1 at the time t_1 given by the equation

$$\frac{2\pi}{T} t_1 - \frac{x_1}{a}\sqrt{\frac{\pi}{T}} + q = \left(2n + \frac{1}{2}\right)\pi$$

where n is any whole number. The maximum temperature at a depth x_2 will be reached at a time t_2 given by the equation

$$\frac{2\pi}{T} t_2 - \frac{x_2}{a}\sqrt{\frac{\pi}{T}} + q = \left(2n + \frac{1}{2}\right)\pi.$$

The minimum temperatures at the same points will be reached when the values are $\left(2n - \frac{1}{2}\right)\pi$.

By subtracting the equations for t_1 and t_2 we get

$$\frac{2\pi}{T}(t_1 - t_2) - \frac{1}{a}(x_1 - x_2)\sqrt{\frac{\pi}{T}} = 0 \tag{14}$$

or

$$\frac{x_2 - x_1}{t_2 - t_1} = 2a\sqrt{\frac{\pi}{T}}. \tag{15}$$

A similar analytical formula can be obtained for the minimum temperatures, and we see that the time at which the maximum or minimum temperature arrives at any depth is later in simple proportion as the depth is greater.

If the points x_1 and x_2 be so chosen that successive maxima or minima occur at them simultaneously, we shall have $t_2 - t_1$ equal to the periodic time T, and the difference $x_2 - x_1$ will be equal to the length of a temperature wave λ, and equation (14) gives

$$2\pi - \frac{1}{a}\lambda\sqrt{\frac{\pi}{T}} = 0$$

or

$$a^2 = \frac{\lambda^2}{4\pi\, T}. \tag{16}$$

In order to determine the diffusivity of the soil it is consequently necessary to measure the wave length λ corresponding to one simple harmonic variation of the known period T, and the conductivity may then be calculated from it. Equation (16) may be written

$$a^2 = \frac{1}{4\pi}\lambda v \tag{17}$$

where v is the velocity of propagation of the temperature wave $\frac{\lambda}{T}$. Hence, the diffusivity is jointly proportional to the wave length and to the velocity of propagation. Again the expression (16) may be written in the forms

$$\lambda^2 = 4a^2 \pi T$$

$$\frac{\lambda^2}{T^2} = 4a^2 \frac{\pi}{T}$$

$$\frac{\lambda}{T} = 2a\sqrt{\frac{\pi}{T}}$$

whence

$$v = 2a\sqrt{\frac{\pi}{T}} \tag{18}$$

which is also derivable from (15) directly.

The velocity of propagation of the temperature wave is directly proportional to the square root of the diffusivity, and inversely as the square root of the period.

Now let us determine the values of the constants A and C in the expression (13). As previously mentioned θ attains its maximum when $\sin \left(\frac{2\pi}{T} t - \frac{x}{a}\sqrt{\frac{\pi}{T}} + q \right)$ is equal to $+1$ and its minimum when this sine is equal to -1. Therefore, for the depth x

$$\begin{aligned} \theta_{\max} &= + A\, e^{-\frac{x}{a}\sqrt{\pi/T}} + C \\ \theta_{\min} &= - A\, e^{-\frac{x}{a}\sqrt{\pi/T}} + C \end{aligned} \right\} \tag{19}$$

whence
$$C = \frac{\theta_{\max} + \theta_{\min}}{2} = \theta_m$$

where θ_m is the mean temperature at the depth x. At the surface $x = 0$ and equation (19) becomes
$$\theta_{\max} = + A + C$$
$$\theta_{\min} = - A + C$$

whence
$$A = \frac{\theta_{\max} - \theta_{\min}}{2} = a_0$$

where a_0 is the amplitude of the temperature variation at the earth's surface. Hence, our equation (13) may be put into the form,

$$\theta = \theta_m + a_0\, e^{-\frac{x}{a}\sqrt{\pi/T}} \sin \left(\frac{2\pi}{T} t - \frac{x}{a}\sqrt{\frac{\pi}{T}} + q \right)$$

or from (16)

$$\theta = \theta_m + a_0\, e^{-\frac{2\pi x}{\lambda}} \sin \left(\frac{2\pi}{T} t - \frac{x}{\lambda} + \frac{q}{2\pi} \right). \tag{20}$$

Thus, the amplitude of the temperature variation at any depth x is jointly proportional to $e^{-\frac{x}{a}\sqrt{\pi/T}}$ or $e^{-\frac{2\pi x}{\lambda}}$ and to the amplitude a_0 of the surface temperature. Or

$$a = a_0\, e^{-\frac{x}{a}\sqrt{\pi/T}} = a_0\, e^{-\frac{2\pi x}{\lambda}} \tag{21}$$

$$\log a = \log a_0 - \frac{x}{a}\sqrt{\frac{\pi}{T}} = \log a_0 - \frac{2\pi x}{\lambda} \tag{22}$$

so that the amplitudes go on decreasing as the heat penetrates downward from the earth's surface; the amplitudes diminish more slowly with depth if the conductivity be higher or if the period be longer, and more quickly if the thermal capacity of the earth be greater. At a certain stratum the amplitudes finally become inappreciable (although they do not vanish except at infinity) and this is called the stratum of invariable temperature. The depth of this stratum may be calculated from the formula (23) as derived from equation (22)

$$x = \frac{\log a_0 - \log a}{2\pi} \lambda. \tag{23}$$

It will be easily seen that the mean temperature θ_m at all depths will not be the same, but will diminish as we go downward from the earth's surface, the curve of mean temperature being logarithmic. The variation of earth temperature at any depth may be represented by a curve, such as fig. 1, which is obtained by plotting the harmonic equation (20), taking x as constant and t as variable. The temperature variations at the various depths at any moment may be represented by a curve, such as fig, 2, which is obtained by plotting the same equation, taking x as variable but t as constant.

A glance at any table of the observed variation of earth temperature will show that the variation is by no means of the simple harmonic character that we have assumed in the above formulæ. There are many sources of disturbance, e. g., the irregularity of the variations of the atmospheric temperature and of surface temperature, heterogeneity of the soil, changes in the cloudiness, the unequal water percolation, etc. If, however, the surface temperature varies to and fro in any
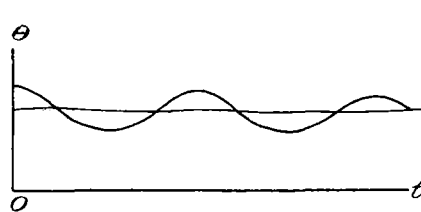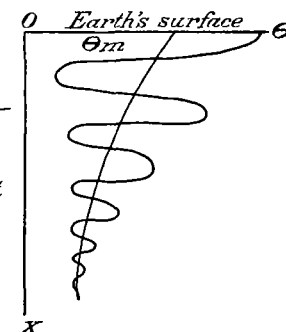

FIG. 1.   FIG. 2.

manner, it may be expressed by a complex harmonic function, such as

$$\theta = \theta_m + \sum_{n=1}^{n=\infty} A_n\, e^{-\frac{x}{a}\sqrt{n\pi/T}} \sin \left(\frac{2\pi}{T} nt - \frac{x}{a}\sqrt{\frac{n\pi}{T}} + q_n \right) \tag{24}$$

which is a general solution of (1), built up of all particular sine solutions, or in more popular form

$$\theta = \theta_m + A_1 e^{-p_1 x} \sin \left(\frac{2\pi}{T} t - p_1 x + q_1 \right) + A_2 e^{-p_2 x} \sin \left(\frac{2\pi}{T} 2t - p_2 x + q_2 \right)$$
$$+ \text{etc.} \tag{25}$$

This is equivalent to the following so-called Bessel's interpolation formula:

$$\theta = \theta_m + a_1 \sin \left(\frac{2\pi}{T} t + \varphi_1 \right) + a_2 \sin \left(\frac{2\pi}{T} 2t + \varphi_2 \right) + a_3 \sin \left(\frac{2\pi}{T} 3t + \varphi_3 \right)$$
$$+ \text{etc.,} \tag{26}$$

which represents the temperature at any time and at any depth; the amplitudes a_1, a_2, a_3, etc., and the phases φ_1, φ_2, φ_3, etc., for any depth may be determined from observational data by means of the method of least squares.

Now the comparison of (22), (23), and (24) offers us the following important relations:

$$\text{Amplitudes} \begin{cases} a_1 = A_1 e^{-p_1 x} \\ a_2 = A_2 e^{-p_2 x} \\ a_3 = A_3 e^{-p_3 x} \end{cases} \tag{27}$$
$$\text{etc.}$$

$$\text{Phases} \begin{cases} \varphi_1 = q_1 - p_1 x \\ \varphi_2 = q_2 - p_2 x \\ \varphi_3 = q_3 - p_3 x \end{cases} \tag{28}$$
$$\text{etc.}$$

$$\begin{bmatrix} p_1 = \sqrt{\dfrac{\pi}{a^2 T}} \\ p_2 = \sqrt{\dfrac{2\pi}{a^2 T}} \\ p_3 = \sqrt{\dfrac{3\pi}{a^2 T}} \end{bmatrix} \text{ or } \begin{bmatrix} a^2 = \dfrac{\pi}{p_1^2 T} \\ a^2 = \dfrac{2\pi}{p_2^2 T} \\ a^2 = \dfrac{3\pi}{p_3^2 T} \end{bmatrix} \tag{29}$$
$$\text{etc.}$$

The diffusivity a^2 of the soil may be calculated in two different ways: The formulæ (27) for the logarithmic decrement of amplitudes, together with the formula (29), give a set of the values of a^2, since the amplitudes a_1, a_2, etc., are given by observations. Another set of the values of a^2 may be obtained by means of the formula (28) for the phase difference, together with the formula (29), since φ_1, φ_2, φ_3, etc., are given by observations, and q_1, q_2, q_3, etc., are phases at the earth's surface.

All the two sets of values of a^2 for any depth would agree perfectly if the data were accurate, and the natural conditions agreed with our hypotheses, e. g., when the soil is homogeneous and possesses uniform specific heat, and when the

isothermal surfaces are parallel planes. As these conditions, however, are not fulfilled, the first thing to find out is how far the different determinations agree and learn accordingly how far the theory may be applied.

(2) PROF. NAKAMURA'S DISCUSSION OF OBSERVATIONS AT TOKYO.

The problem of the variation of the soil temperature has attracted the attention of Japanese meteorologists and all the meteorological stations of the first order are now equipped with earth thermometers. The first paper on the subject that has come to my notice is one published in Japanese in the Journal of the Tokyo Physico-Chemical Institute, in 1893, by Prof. K. Nakamura, now director of the Central Meteorological Observatory of Japan. His discussion is based on the observations taken at Tokyo during the three years 1889–1892 at the observatory whose geographical coordinates are:

Latitude = 35° 41′ north.

Longitude = 139° 45′ east.

Altitude = 20 meters above sea level.

The earth thermometers were placed at the depths 0.0, 0.3, 0.6, 1.2, 3.0, 5.0, and 7.0 meters. From the tables given by Doctor Nakamura the following results of observations have been copied as regards the times of the occurrence of the annual maximum and minimum temperatures at various depths.

TABLE 1.

Depth, in meters.	Epoch of occurrence of —	
	Maximum.	Minimum.
0.0	August (II)	January (III)
0.3	August (III)	February (I)
0.6	August (III)	February (II)
1.2	September (II)	March (I)
3.0	November (II)	May (I)
5.0	January (III)	August (I)
7.0	April (II)	October (II)

NOTE.—I indicates the first, II the second, and III the third decade of the month.

It will be seen that the maximum and minimum temperatures occur on the earth's surface in August and January, respectively, but at a depth of five meters they occur in January and August, respectively, or one-half year behind.

Next Doctor Nakamura analyzed his observational data by the formula (26), taking the first three terms only and computed the values of the amplitudes a_1, a_2, a_3, and the phases φ_1, φ_2, φ_3, as in Table 2.

TABLE 2.

Depth in meters.	Mean temperature. θ_m	Amplitudes.			Phases.		
		a_1	a_2	a_3	ϕ_1	ϕ_2	ϕ_3
	° C.	° C.	° C.	° C.	°	°	°
0.0	15.8	12.64	1.54	0.50	247.9	301.1	137.7
0.3	16.3	10.31	1.04	0.29	237.9	281.9	107.3
0.6	16.0	9.02	0.82	0.31	228.8	273.1	81.0
1.2	16.4	6.53	0.51	0.19	208.4	245.6	15.2
3.0	15.7	2.64	0.19	0.11	147.5	160.5	259.5
5.0	15.5	0.70	0.01	0.02	63.1	222.2	120.3
7.0	15.3	0.22	0.02	-0.01	357.6	259.9	83.7

The author then computed the temperatures for each portion of a year by means of these constants, and found that the observational and computed values agree quite closely. Next substituting the values of a and φ, from Table 2, in the formulæ (27) and (28), Nakamura determined the values of the constants A, p, and q from which he formed the following equations:

$$a_1 = 12.856\, e^{-0.00573\,x}$$

$$a_2 = \ldots\ldots$$

$$\varphi_1 = 250.2° - 0.363°\, x$$

$$\varphi_2 = 302.0° - 0.520°\, x$$

etc.

With these equations he again computed the values of a and φ for different depths which agree well with those in Table 2, though we find that the discrepancies are rather greater in the case of a_2, a_3, and φ_3.

Finally the author computed the values of a^2. According to his computation

$$p_1 = 0.00573$$

$$p_2 = 0.00907$$

whence by equation (29) we have (in C. G. S. units) from p_1

$$a^2 = 0.00246$$

and from p_2

$$a^2 = 0.00263$$

the mean of which is

$$a^2 = 0.00250$$

(3) DOCTOR OISHI'S DISCUSSION OF OBSERVATIONS AT TOKYO.

The second paper[4] on earth temperature at Tokyo was published in 1904 by Dr. W. Oishi, Meteorologist of the Central Meteorological Observatory. This discussion is based on the observations of more than fifteen years, (1886–1902). The earth thermometers were planted in an open, flat field whose soil consisted of loam, its surface being covered with short grass. The depths and other data are given in Table 3.

TABLE 3.

Depths below surface.	Thermometers used.	Number of observations.
Meters.		
(1) 0.0	Ordinary mercurial thermometer laid on the ground, with its bulb covered with soil.	Hourly.
(2) 0.05 (3) 0.1 (4) 0.2	Bulbs buried at respective depths and stems exposed in the shade.	6 times daily.
(5) 0.3 (6) 0.6 (7) 1.2 (8) 3.0 (9) 5.0 (10) 7.0	Slow-acting thermometers, suspended by chains in iron tubes, 2.7 centimeters in diameter; sealed at lower ends and closed above the ground with copper cups.	Twice daily, 10 a.m. and 10 p.m. Once a day, at 10 a.m.

Doctor Oishi discusses the diurnal, annual, and secular variations of earth temperature and gives tables of the temperatures of the air, the surface, and the earth. Comparing the values of air temperature at 1.2 meters above the ground with the temperature of the earth's surface, the author says that the monthly mean surface temperature is generally higher than the air temperature, and in summer the difference is very remarkable; in winter the difference is less and the surface temperature is lower. Oishi thinks it very strange that the surface temperature is lower than the air temperature during daytime in winter, while it is expected to be higher in ordinary cases. For the explanation of these facts he mentioned that the surface was protected from sun's heat by short grass, and, moreover, it was moist and there was much evaporation from it. While the daily maximum of air temperature occurs at 2 p. m. through the whole year, that of the surface temperature takes place at 1 p. m., except in winter, during which it occurs at 2 p. m. The daily minima of surface and air temperatures both occur at 5 a. m. in summer and 7 a. m. in winter.

The comparison of the surface and the deeper earth temperatures shows that the diurnal variation is remarkable on the surface, but it steadily diminishes with increasing depth, and the times of occurrence of maxima and minima are gradually retarded as anticipated in the mathematical theory. For example, the author shows that in January the temperature range amounts to 6.8° C. on the surface, while it is reduced to 0.1° at the depth of 0.3 meter; in July it is 11.4° on the surface, but 0.2° at that same depth. The daily maximum temperature of January occurs at 2 p. m. on the surface, while at 0.3 meter

[4] The Bulletin of the Central Meteorological Observatory of Japan, No. 1.

it is retarded to 2 a. m. of the next day. In July, the surface maximum temperature is observed at 1 p. m., while the maximum at 6.3 meters occurs at 10 a. m. Minimum temperatures of January occur at 7 a. m. on the surface and at 6 p. m. at a depth of 0.3 meter, and those of July take place at 5 a. m. and noon, respectively.

Doctor Oishi then furnishes us a very important table of 5-day means of earth temperatures for the year. As in the case of daily temperature range, so also the annual range decreases rapidly under the ground, and the times of occurrence of maximum and minimum temperatures are retarded gradually.

TABLE 4.

Depth, in meters.	Annual range.	Time of occurrence.	
		Max.	Min.
	°		
0.0	28.2	Aug. 11	Jan. 23
0.3	22.7	Aug. 16	Jan. 23
0.6	18.7	Aug. 21	Jan. 31
1.2	14.0	Sept. 15	Feb. 27
3.0	5.2	Nov. 6	May 3
5.0	1.3	Feb. 2	July 30
7.0	0.4	April 30	Oct. 30

The author now applies to the 5-day means of the earth temperatures the formula (26) as far as the fourth term. By means of the method of least squares he determined the coefficients a_1, a_2, a_3, a_4 and φ_1, φ_2, φ_3, φ_4, for each depth separately as given in Table 5.

TABLE 5.

Depth, in meters.	θ_m	a_1	a_2	a_3	a_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4
	° C.	° C.	° C.	° C.	° C.	°	°	°	°
0.0	15.629	12.931	1.155	0.641	0.485	244.55	298.05	136.26	290.79
0.3	15.713	10.733	0.863	0.543	0.313	235.76	284.08	133.13	228.84
0.6	15.693	9.051	0.692	0.403	0.209	225.47	279.82	69.49	270.54
1.2	15.958	6.696	0.383	0.197	0.056	206.03	261.89	32.96	271.02
3.0	15.779	2.514	0.112	0.085	0.025	142.77	174.34	291.43	43.35
5.0	15.499	0.640	0.011	0.004	0.007	59.07	142.13	104.04	60.26
7.0	15.390	0.192	0.009	0.008	0.002	341.19	200.56	345.96	63.44

As did Nakamura so also Oishi substituted these values for the constants of the formula (26) and computed the values of θ for each epoch and each depth; the computed values agree reasonably with the observed data.

The most important feature of the present paper is perhaps the two independent computations of the values of the soil diffusivity. According to Oishi the values of a^2_a, calculated from amplitudes and a^2_ϕ from phase differences are as in Table 6.

TABLE 6.—Coefficients of temperature diffusivity of the soil at different depths in C. G. S. units.

Depth, in meters.	a^2_a	a^2_ϕ	$\frac{1}{2}(a^2_a + a^2_\phi)$
0.0—0.3	0.00259	0.00381	0.00320
0.3—0.6	0.00307	0.00278	0.00293
0.6—1.2	0.00395	0.00311	0.00353
1.2—3.0	0.00336	0.00265	0.00301
3.0—5.0	0.00213	0.00186	0.00200
5.0—7.0	0.00275	0.00216	0.00246
Average..	0.00298	0.00273	0.00286

The author shows how closely these values agree with those calculated from the equation for amplitudes, and from the equation for phase differences, by finding the constants only for the first term, a_1, p_1, and q_1.

$$a^2_a = 0.00276$$
$$a^2_\phi = 0.00230$$
$$\tfrac{1}{2}(a^2_a + a^2_\phi) = 0.00253$$

This value holds, of course, in the case of simple harmonic variation of temperature only. From the amplitude equation

$$a_x = 13.347\, e^{-0.006009\,x}$$

the annual temperature range at Tokyo is shown to reduce to 0.2° C. at the depth of 8 meters and to 0.01° C. at 13 meters. Thus, a great difference is observed in the depths at which the annual and diurnal ranges are respectively reduced to a given fraction of the same range at the surface. Theoretically the depths are as the square roots of the periods; i. e., as $\sqrt{365.24}$ to 1, or nearly 19 to 1. The author concludes that this fairly agrees with the results of observations.

Finally, Oishi gives an interesting account of the secular variations of the temperatures of the air and earth. According to a table prepared by him the secular changes of temperature observed in the air are noticed under ground, and generally in the same year down to a depth of 1.2 meters, but in the following year at the strata from 3 to 7 meters below the surface. Thus, the maximum temperature (14.77° C.) in the air in 1894 was observed also down to the depth of 1.2 meters in the same year, and at the depths of 3, 5, and 7 meters in the same year. The maximum temperature of 1898 was observed at 0.3 meter below in the same year; at the depths 0.6 to 5 meters, in the following year; and at the depth of 7 meters in 1900. The minimum temperature in 1897 occurred at the depth of 1.2 meters in the same year, at the depths 3 to 5 meters in the following year, and 7 meters in 1898 and 1899.

OKADA'S DISCUSSION OF THE OBSERVATIONS AT NAGOYA.

Dr. T. Okada, of the Central Meteorological Observatory, has published two papers on this subject, with the titles, "On the Underground Temperature Observations made at Nagoya, Japan,"[5] and "Discussion of the Earth Temperature Observations made at Osaka Meteorological Observatory."[6] The data for the first paper consist of the record of ten years' observations (1894-1903) at Nagoya Meteorological Observatory, which is situated in 35° 10' north and 136° 55' east, and lies fifteen meters above sea level. The soil in which the observations were made consists of a mixture of sand and loam, and the surface is covered with sod. The temperatures were observed at the depths of 0.0, 0.3, 1.5, 3.0, 6.0, and 12.0 meters. The instruments and methods employed were about the same as those described by Oishi.

According to Okada's table, the mean temperature of the earth's surface is higher than that of the air throughout the year. The difference is greatest in August and least in January, and is about 2° in the annual mean. The epochs of occurrences of the extreme temperatures are gradually retarded with increasing depth, as shown in Table 7, but do not quite agree with those observed by Doctors Nakamura and Oishi at Tokyo.

TABLE 7.

Depth, in meters.	Annual range.	Time of occurrence.	
		Max.	Min.
	° C.		
0.0	25.5	August.	January.
3.0	8.3	October.	April.
6.0	2.0	December.	June.

The constants of the harmonic formula (26) as computed by Okada are as in Table 8.

TABLE 8.

Depth, in meters.	θ	a_1	a_2	a_3	ϕ_1	ϕ_2	ϕ_3
	° C.	° C.	° C.	° C.	°	°	°
0.0	16.56	12.42	1.06	0.60	244.1	307.2	116.1
0.3	16.50	10.46	0.88	0.43	235.2	290.9	111.1
1.5	16.47	6.59	0.40	0.17	198.7	280.3	57.3
3.0	16.42	4.00	0.26	0.16	160.1	278.1	317.7
6.0	15.85	1.03	0.17	0.03	100.5	26.7	322.5
12.0	15.75	0.02	0.01	0.02	181.4	341.5	183.2

[5] Journal of the Meteorological Society of Japan, 1904.   [6] Ibid, 1905.

Values for the diffusivity of the soil are given in Table 9.

TABLE 9.

[In C. G. S. units.]

Depth, in meters.	$a^2 {}_a$	$a^2 {}_\phi$	$\frac{1}{2}(a^2 {}_a + a^2 {}_\phi)$
0. 0–0. 3	0. 00304	0. 00372	0. 00338
0. 3–1. 5	0. 00672	0. 00353	0. 00512
1. 5–3. 0	0. 00899	0. 00494	0. 00696
3. 0–6. 0	0. 00487	.0. 00829	0. 00658

The author places more reliance on the values computed from the change of amplitudes than those obtained from the retardation of phase. The soil at a depth from 1.5 to 3.0 meters appears to possess the highest value of diffusivity.

Okada then computed, by means of the formula (18), the velocity of the annual temperature wave as it is propagated from the surface to the interior.

TABLE 10.

Depth, in meters.	Velocity, cm. per day.
0. 0–0. 3	3. 4
0. 3–1. 5	3. 3
1. 5–3. 0	3. 9
3. 0–6. 0	5. 0
Mean	3. 9

Thus the velocity apparently increases with the depth and on the average would amount to $365 \times 3.9 \div 100 = 14.2$ meters per year if we could suppose that the soil were uniform to that depth.

The logarithmic equation for the amplitude at any depth (see equation 21), was found to be

$$a_x = 12.43\, e^{-0.0040884\,x}$$

which gives values for amplitudes fairly close to the observational values. By this formula the depth of the stratum of invariable temperature at Nagoya was calculated. It was found that the annual amplitude of temperature is reduced to 0.5° C at a depth of 7.8 meters and to 0.1° C at 11.8 meters. Hence the invariable stratum may be fairly assumed to lie at the depth of 12 meters at this place.

Finally, adopting von Bezold's formula[7], Doctor Okada determined the total amount of annual heat exchange in the ground at Nagoya to be 47.6 calories per square centimeter of ground surface, which is very close to that for Nukuss obtained by von Bezold himself.

(5) OKADA'S DISCUSSION OF OBSERVATIONS AT OSAKA.

Doctor Okada's second paper contains an analysis of the earth temperature data deduced from ten years observations (1895–1904) at Osaka, which is situated in latitude 34° 42' north, and longitude 135° 31' east from Greenwich and is five meters above sea level.

The ground in which the observations were made consisted chiefly of granitic sand, and the surface was flat and completely covered with sod. The depths at which the thermometers were placed were 0.0, 0.3, 1.2, 3.0 and 5.0 meters.

For the determination of the water capacity of the soil the author selected two samples of the soil; one was taken from the stratum 0.3 meter below the surface, and the other from

[7] If θ is the temperature at the depth x, C the heat capacity, H the depth of the stratum of invariable temperature, then the amount of heat which is introduced to the soil per unit area in the time during which the temperature increases from 0° to θ° is

$$Q = \int_0^H C\theta\, dx.$$

The difference between the maximum and minimum values of Q is the amount of heat exchange during the year. W. von Bezold, Der Wärmeaustausch. Berlin, 1902.

the depth of one meter by boring. From the measured loss of weight by thorough drying it was found that the water content of the soil is practically the same, and average 0.074 gram per cubic centimeter. Of course the capacity is variable with the season, or strictly speaking, it varies from day to day.

The determination of the specific heat of pulverized matter, like soil constituents, is certainly a difficult task in practical physics. By the method of mixture Okada roughly determined the specific heat of the dry soil, the result being as follows:

TABLE 11.

No.	Specific heat.
I	0. 203
II	0. 203
III	0. 196
IV	0. 213
V	0. 204
Mean	0. 204

At Osaka the minimum temperature at the earth's surface occurs in January and the maximum in August; the minimum in February and the maximum in August at the depth of 1.2 meters; the minimum in May and the maximum in November at the depth of 5.0 meters. The annual mean temperature increases down to the depth of 3.0 meters and then decreases with depth.

TABLE 12.—The constants of the harmonic formula (26) for Osaka.

Depth, in meters.	θ	a_1	a_2	a_3	ϕ_1	ϕ_2	ϕ_3
	°C.	°C.	°C.	°C.	° '	° '	° '
0. 0	16. 93	13. 320	1. 539	1038	261 06	344 18	164 32
0. 3	16. 84	11. 880	1. 020	0628	255 44	342 11	151 09
1. 2	17. 40	8. 578	0. 572	0369	235 09	326 42	120 02
3. 0	18. 05	4. 188	0. 119	0101	194 54	305 25	10 12
5. 0	17. 70	1. 603	0. 073	0029	143 40	255 01	306 08

The amplitude of the variation rapidly decreases with depth and the retardation of phase is approximately proportional to the depth if we leave that of the stratum at 5.0 meters out of consideration. The amplitude of annual variation of the underground temperature becomes very small with increasing depth, so that in general the determination of phase is very inaccurate.

Thus, the law that the retardation of phase is proportional to the depth, holds only approximately in natural soil owing largely to changes of structure and water content.

Doctor Okada obtained for an amplitude at any depth, x, the following equation:

$$\log a = \log (13.76) - 0.001825\,x.$$

By this formula it is easily known that the range of annual variation of temperature becomes 1° at the depth of 8 meters and 0.5 at 9.5 meters, and also that the annual amplitude practically vanishes at 12 meters below the earth's surface.

The values of the diffusivity of the soil at Osaka as obtained by Okada are given in Table 13.

TABLE 13.

Depth.	$a^2 {}_a$	$a^2 {}_\phi$	$\frac{1}{2}(a^2 {}_a + a^2 {}_\phi)$	k
0. 0–0. 3	0. 00683	0. 01024	0. 00853	0. 00341
0. 3–1. 2	0. 00760	0. 00626	0. 00693	0. 00277
1. 2–3. 0	0. 00629	0. 00655	0. 00642	0. 00257
3. 0–5. 0	0. 00432	0. 00499	0. 00465	0. 00186

The values of diffusivity deduced from the variation of amplitude and from the difference of phase agree fairly with each other, if we neglect the case of the earth's surface and also that of the stratum at the depth 0.3 meter.

Next, the values of diffusivity at Tokyo, Nagoya, and Osaka were compared, as in Table 14.

TABLE 14.

Place.	Soil.	0.0—0.3	0.3—1.2	1.2—3.0	3.0—5.0
Osaka ...	Sand...........	0.00853	0.00693	0.00642	0.00465
Tokyo ...	Loam...........	0.00320	....	0.00301	0.00200
Nagoya..	Sand and loam..	0.00338	0.00512	0.00696	0.00658

At Osaka and Tokyo the diffusivity seemingly diminishes as we go into the deeper strata, while at Nagoya it increases with depth down to a certain stratum. The diffusivity of the upper layer at Nagoya is equal to that at Tokyo, while the diffusivity of the deeper stratum rather resembles that at Osaka. The reason for this, according Doctor Okada, is that the soil at Osaka is composed of granitic sand, and that at Tokyo of loam, while at Nagoya it is not uniform, the upper stratum being composed of loam and the deeper of sand with some loam. The conductivity of the Osaka soil decreases with depth, the mean conductivity being 0.00265 in C. G. S. units. Thus, the conductivity of sandy soil is one hundredth part of that of zinc, and is nearly equal to that of glass, while it is as much as ten times the conductivity of snow that has a density 0.18.

Applying Schubert's formula,[8] Doctor Okada computed the total amount of annual heat exchange to be 2163 gram-calories per unit area at the stratum of 4.0 meters below the surface. According to Angot the total annual amount of insolation at the latitude of 35° is 220018 gram-calories per square centimeter assuming the coefficient of transparency as 0.7 and the solar constant at 3 gram-calories per square centimeter per minute. Hence, the author concludes that the quantity of solar energy consumed in heating the soil is one-hundredth part of the total insolation which is received by the surface of the soil.

AN ACCOUNT OF RECENT METEOROLOGICAL AND GEO-PHYSICAL RESEARCHES IN JAPAN.

By Dr. S. TETSU TAMURA, Washington, D. C. Dated August 30, 1905.

The important scientific papers published in Japan are usually found in the Journal of the College of Science of the Tokyo Imperial University, in the Proceedings of the Tokyo Physico-Mathematical Society, or in the Journal of the Meteorological Society of Japan. I find, however, that neither of these publications are easily accessible in America, except in large Government bureaus or in prosperous university libraries. Moreover, those papers published elsewhere in Japanese are in no way accessible or intelligible to the western scientists. Perhaps this may be one of the reasons why Japanese scientific works are not duly known to American laymen as well as to scientists, though very brief references are given in "Science Abstracts". In the preceding article I have given an account of earth temperature investigations in Japan, and I am now induced to give some reviews, with occasional notes, of other memoirs by Japanese scientists on meteorology and its allied sciences.

1. PROF. F. OMORI ON THE ANNUAL VARIATION OF THE HEIGHT OF SEA LEVEL.

Professor Omori, of the Tokyo Imperial University, has published two papers on this subject in the Proceedings of the Tokyo Physico-Mathematical Society, Vol. II, Nos. 13 [1904], and 20 [1905]. The interesting feature of these papers is the comparison of the variation of the height of sea level with that of barometric pressure. In the first paper the author discussed their annual variations at Ayukawa, in the Province of Rikuzen, and Misaki, in the Province of Sagami, which two places are situated on the Pacific coast.

The following table gives the mean monthly relative heights of the sea level at Ayukawa and Misaki, deduced from the

8 Zur Theorie der Wärmeleitung im Erdboden. Phys. Zeitschr, 1901. I Jahrgang, No. 41, p. 444.

tide-gage observations during the year 1902, and also the mean barometric pressure in the vicinities of these places.

TABLE 1.

Month.	Monthly mean height of sea level. Ayukawa.	Monthly mean height of sea level. Misaki.	Barometric pressure, reduced to sea level.
	mm.	mm.	mm.
I	121	129	761.3
II	0	0	764.5
III	23	69	762.2
IV	45	103	760.2
V	96	188	759.4
VI	171	171	756.1
VII	197	175	766.2
VIII	184	226	758.8
IX	219	276	758.7
X	172	246	764.1
XI	117	164	765.5
XII	157	257	761.0

From Table 1 it will be seen that the monthly variations of sea-level heights at Misaki and Ayukawa are quite similar, but just the reverse of the variations of barometric pressure. The range of the barometric variation is 9.3 mm., which corresponds to 9.3 × 13.6 = 126 mm., height of water. On the other hand, the range of the mean monthly variation of height of sea level is 276 mm. at Misaki and 219 mm., at Ayukawa, or twice as large as the range of the barometric variation. Hence, Professor Omori concludes that the sea bottom is subjected to a greater total pressure in the summer months than in February, March, and April. The cause of this elevation of the sea level in the summer the author attributes partly to the decrease of barometric pressure in Japan and its vicinities, and partly to the existence of the high pressure over the North Pacific Ocean, which causes the depression of the sea level in that region. In the winter the phenomena are just the reverse. From these facts Professor Omori explains why there are at the bottom of the west Pacific Ocean more earthquakes in the summer than in the winter.

The second paper by F. Omori contains the discussions of similar data obtained for four different places on the coast of the Japan Sea. The places are:

1. Hamada, in the Province of Iwami.
2. Wajima, on the northern coast of the Peninsula of Noto.
3. Iwasaki, on the western coast of the Province of Mutsu.
4. Otaru, in the Province of Shiribeshi, Hokkaido.

The mean monthly relative heights of the sea level at the four different places during 1902, and the monthly means of barometric pressure at the same places are given in Table 2.

TABLE 2.

Month.	Mean height of sea level at the four places.	Mean height of barometer at same places.
		mm.
I	112	761.38
II	0	764.20
III	48	761.61
IV	88	758.85
V	191	757.70
VI	206	757.21
VII	236	755.44
VIII	276	757.49
IX	312	757.55
X	257	763.64
XI	176	764.42
XII	181	761.37

From this table the annual variation of the barometric pressure will be seen, as in the case of the places on the Pacific coast, to be nearly the reverse of that of the height of sea level. Now the range of the former was 8.98 mm. which corresponds to 8.98 × 13.6 = 122 of water. The range of the variation of sea-level heights was 312 mm. Along the coast of the Japan Sea the variation of the sea-level height is opposite